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MEMORANDUM REPORT BRL-MR-3953

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A COMPUTER PROGRAM TO REDUCE
DIGITIZED YAW-CARD DATA FROM
FINNED KINETIC-ENERGY PROJECTILES

RICHARD A. PENNEKAMP

DECEMBER 1991

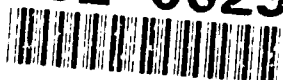
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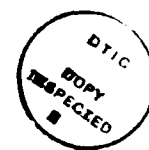
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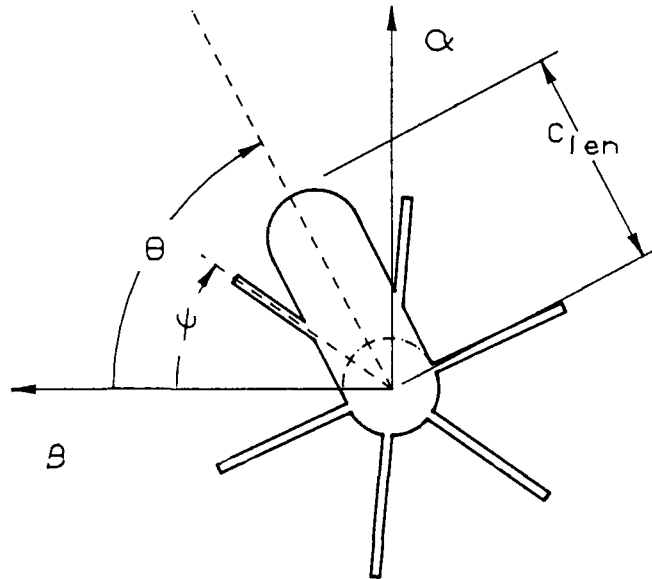
1. INTRODUCTION

Witness panels, commonly referred to as yaw cards, are used in ballistic testing to determine the pitch (α) and yaw (β) of a projectile as a function of range. Yaw cards are traditionally measured by a device called a yaw-card template. This device is calibrated to measure the total angle of attack, δ , where δ is related to α and β :

$$\alpha = \delta \sin \theta \quad (1)$$

$$\beta = \delta \cos \theta \quad (2)$$

The angle θ is defined in Figure 1. In order to obtain θ , an instrument that measures angles (usually a protractor) must be utilized. The α and β data are usually input to some computer program: for example, to determine flight characteristics of a projectile.



“AS SEEN FROM THE MUZZLE”

Figure 1. Yaw-Card Coordinate System.

The aforementioned technique for measuring yaw-cards is adequate, but requires manually entering data into a computer after measuring two quantities with two different tools. A method that uses an electronic digitizer has been developed to quicken the data-reduction process. It is a procedure that acquires data that can be used to calculate α , β , and the fin orientation angle ψ . The ψ value is used to determine the spin rate as a function of range (Pennekamp 1990). The data obtained by the digitizer can either be processed by the

digitizer's computer or transferred to another computer for processing. This eliminates the necessity of manually entering the data into the computer.

This report describes how the digitized data are manipulated to obtain the required results. Appendix A has a listing of a FORTRAN program that aids in the reduction of the yaw-card data. An example of the use of the the program is given in Appendix B.

2. THEORY

In general, yaw cards are used to determine the δ value of a projectile at a given range location. The δ value has the following relationship with the hole in the yaw card:

$$C_{len} = r \cos \delta + \ell_p \sin \delta, \quad (3)$$

where C_{len} is defined in Figure 1, r is the radius of the flight body, and ℓ_p is the length of the cylindrical portion of the flight projectile (see Figure 2). Appendix C describes how ℓ_p is determined. If small angles ($< 10^\circ$) are assumed, Equation 3 simplifies to

$$\delta \approx \sin^{-1} \left(\frac{C_{len} - r}{\ell_p} \right). \quad (4)$$

It should be noted that $\theta_c > \delta$ for Equation 4 to be valid.

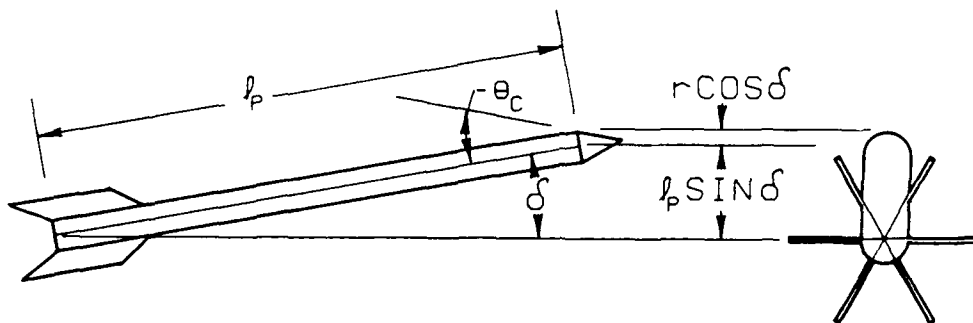


Figure 2. Relationship between the Yaw-Card Hole and δ .

The only unknown value in Equation 4 is C_{len} . As seen in Figure 3, C_{len} is the distance between P1 and P2. If the coordinates of these two points were known, C_{len} could be

determined by

$$C_{len} = \sqrt{(P1_x - P2_x)^2 + (P1_y - P2_y)^2}, \quad (5)$$

where the subscripts denote either the "x" or "y" value in the traditional Cartesian coordinate system.

A digitizer is a machine that provides the coordinates of a user-selected point. The user places a component of the digitizer on the point and presses a button. A signal is sent to the computer portion of the digitizer where the coordinates of the point are determined. This process will be referred to as "digitizing" a point.

With the use of the digitizer, the locations of P1 and P2 can be determined in the XY plane. The location of P2 is easy to acquire; just locate the extreme edge of the yaw card hole and digitize the point. P1 cannot be determined in this straightforward manner. P1 is the center of a circle whose diameter is the fin span. The extreme edges of the fin blade imprints (points P3-P8) lie on the circumference of this circle. (In reality it is an ellipse, but for small angles a circle is a reasonable approximation.) The points P3 through P8 can be digitized, and they must satisfy the equation of a circle in the XY coordinate system (Beyer 1981),

$$R_{fin}^2 = (P1_x - Pn_x)^2 + (P1_y - Pn_y)^2 \quad (6)$$

where R_{fin} is the radius of the fin span, and n has the value of 3 through 8. The only unknown values in Equation 6 are the x and y values of P1, and these can be determined by applying nonlinear least squares (Scarborough 1930) to Equation 6. The nonlinear least squares technique is iterative and requires initial guesses to start the process: reasonable estimates for the x and y values of P1. They can be provided by digitizing a point that is near P1, for example P9 in Figure 3. After a solution to the nonlinear least-squares technique is obtained, the location of P2 is known. This allows C_{len} to be determined (Equation 5) and from that information, δ can be obtained (Equation 4).

In order to calculate α and β , the angle θ must be determined. The angle θ is given by the relation

$$\theta = \phi + 90^\circ, \quad (7)$$

where ϕ is the angle between the reference line and the line that contains P1 and P2 (see Figure 3). The points P10 and P11 are on the reference line and their x and y values can be obtained with the digitizer. The x and y values of P1 and P2 have already been obtained in order to calculate δ . With the x and y values for points P1, P2, P10 and P11 known, the direction cosines, A and B, can be calculated from the following formulas (Beyer 1981):

$$A_1 = \frac{P2_x - P1_x}{d_{2-1}} \quad (8)$$

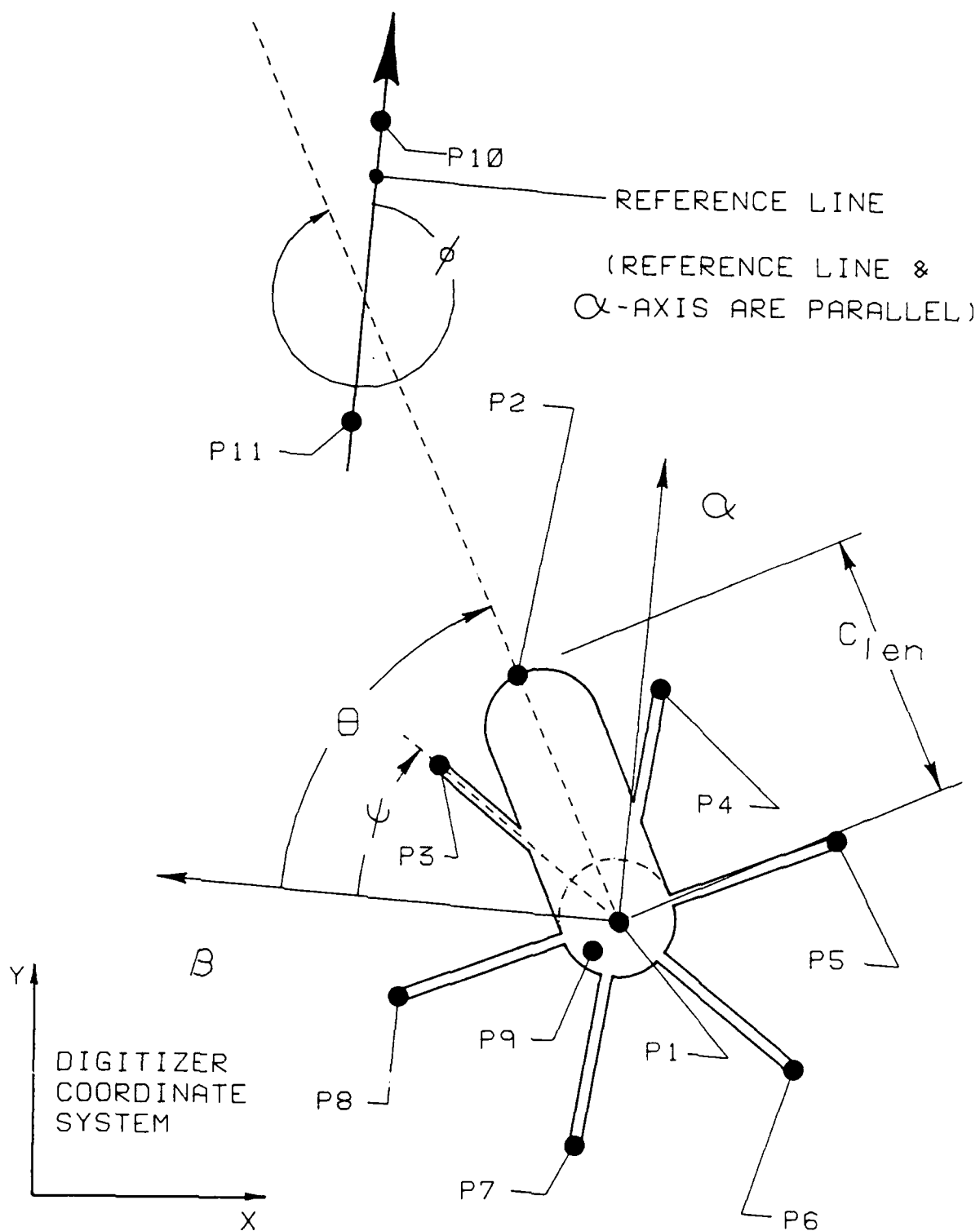


Figure 3. Digitizer Coordinate System.

$$B_1 = \frac{P2_y - P1_y}{d_{2-1}} \quad (9)$$

$$A_2 = \frac{P11_x - P10_x}{d_{11-10}} \quad (10)$$

$$B_2 = \frac{P11_y - P10_y}{d_{11-10}} \quad (11)$$

where d_{2-1} is the distance between P2 and P1 and can be calculated by using Equation 5 ($d_{2-1}=C_{len}$). The d_{11-10} term is defined in a similar fashion. With the direction cosines known, an angle ϕ^* can be determined by (Beyer 1981):

$$\phi^* = \cos^{-1}(A_1 A_2 + B_1 B_2). \quad (12)$$

Only the principle value of the arc cosine is computed; that is,

$$0^\circ \leq \phi^* \leq 180^\circ. \quad (13)$$

If the angle ϕ (as shown in Figure 3) is less than 180° , then

$$\phi = \phi^*; \quad (14)$$

and if not, then

$$\phi = 360^\circ - \phi^*. \quad (15)$$

A method to determine if ϕ is larger or smaller than 180° will now be discussed.

The first step in determining if ϕ is larger or smaller than 180° requires that the slope of the reference line be calculated by the following equation:

$$\Delta_{ref} = \frac{P10_y - P11_y}{P10_x - P11_x} \quad (16)$$

The slope from Equation 16 (Δ_{ref}) can be infinity (∞), zero, positive or negative.

The first case examined is $\Delta_{ref}=\infty$ (this occurs when $P10_x=P11_x$). The ϕ angle is greater than 180° if $P1_x$ is greater than $P2_x$. If this is the case, then Equations 12 and 15 are used to calculate ϕ . If not, Equations 12 and 14 are used to determine ϕ .

For all the other possible values of Δ_{ref} , the equation of the line that goes through the Point P1 and has the slope of Δ_{ref} must be calculated. This is accomplished by determining the y-intercept (b_{ref}) of the line from the following equation:

$$b_{ref} = P1_y - \Delta_{ref} P1_x \quad (17)$$

For Δ_{ref} zero or positive, the line with the slope of Δ_{ref} and the y-intercept of b_{ref} can be used to establish the following inequality:

$$y < \Delta_{ref} x + b_{ref}. \quad (18)$$

If P2's x and y values do not satisfy the above inequality, then ϕ is determined by using Equations 12 and 15. If they do, then ϕ is calculated by using Equations 12 and 14.

For Δ_{ref} negative, the following inequality is utilized:

$$y \geq \Delta_{ref}x + b_{ref}. \quad (19)$$

Again, if P2's x and y values do not satisfy the above inequality, then ϕ is determined by using Equations 12 and 15. If they do, then ϕ is calculated by using Equations 12 and 14.

From this information, θ can be calculated from Equation 7, and α and β can be calculated by using Equations 1 and 2.

Another bit of information that can be obtained from a yaw card is the fin orientation angle, ψ . The locations of all the points (P1, P3, P10 and P11) required to determine ψ would have to be obtained in order to calculate δ and θ . With this information, the direction cosines can be calculated, and ψ can be determined in the same fashion as ϕ .

3. CONCLUSION

A method of measuring the data from a yaw card using an electronic digitizer has been described. It requires digitizing a finite number of points (≈ 10) on each card, and from that information, the total angle of attack, the pitch, the yaw, and the fin orientation angle can be obtained. The process produces data on computer files, which eliminates manually entering the data.

The results from the digitized readings compare well with the results from reading the yaw cards with the template and protractor. Some 213 yaw cards were read with both methods, and the average difference for the δ readings was less than 0.05° . For the ψ values, the average difference was 0.2° . (The θ values were not read with the protractor, but their results are assumed to be similar to the ψ values.) These results suggest that no accuracy is lost by using the digitizer to read the yaw cards.

Because there is no loss in accuracy and a potential for time savings, the utilization of a digitizer to aid in the reduction of yaw cards is a logical step in the modernization of ballistic-data reduction.

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- Pennekamp, R.A., "Yaw and Spin Characteristics of the XM900E1 Model 545 APFSDS-T Projectile," Ballistic Research Laboratory, Aberdeen Proving Ground, MD, Memorandum Report BRL-MR-3846, June 1990. (AD B145808)
- Scarborough, J.B., "Numerical Mathematical Analysis," pp.374-376, George Banta Publishing Co., Menasha, WI, 1930.

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APPENDIX A:
LISTING OF A FORTRAN 77 PROGRAM THAT REDUCES DIGITIZED
YAW-CARD DATA

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```

        dimension x(10),y(10)
        character*72 describ
*****
*File 3 contains pertinent projectile information
*****
        open(3,file='rdtype',status='old')
        read(3,101)describ
        read(3,*)finsp
        read(3,*)plen
        read(3,*)prad
        read(3,*)finno
        close(3)
*****
        radfin=finsp/2.
        j=1
*****
*File 1 contains the digitized data, File 2 is the output file
*****
        open(1,file='digdat',status='old')
        open(2,file='output',status='new')
        write(2,101)describ
        write(2,103)
*
*-----
*Loop 5  reads the data from the DIGDAT file.
*
5      read(1,*,end=6)xr2,yr2
      read(1,*)xr1,yr1
      i=1
2      read(1,*)x(i),y(i)
*****
*This statement looks for the switch that signals the end of fin-imprints
*entries. It has been arbitrarily set at (3,3).
*****
      if((x(i).lt.3).and.(y(i).lt.3))goto1
      i=i+1
      goto2

```

```

1      numpts=i-1
      read(1,*)xc,yc
      read(1,*)xdelta,ydelta
      call circen(x,y,numpts,xc,yc,radfin)
      pi=2*asin(1.)
*****
*phi and theta calculations
*****
      call angle(xr1,yr1,xr2,yr2,ca1,cb1)
      call angle(xc,yc,xdelta,ydelta,ca2,cb2)
      phi=acos(ca1*ca2+cb1*cb2)
      phi=phi*180/pi
      call truang(xc,yc,yr2,xr2,yr1,xr1,xdelta,ydelta,phi)
      theta=phi+90.
      if(theta.gt.360.)theta=theta-360.
*****
*psi calculations
*****
      call angle(xc,yc,x(1),y(1),ca2,cb2)
      psi=acos(ca1*ca2+cb1*cb2)
      psi=psi*180/pi
      call truang(xc,yc,yr2,xr2,yr1,xr1,x(1),y(1),psi)
      psi=psi+90.
      spaang=360./finno
4      if((psi.ge.0.).and.(psi.le.spaang))goto3
      psi=psi-spaang
      goto4
*****
*delta, alpha and beta calculations
*****
3      clen=sqrt((xc-xdelta)**2+(yc-ydelta)**2)
      delta=asin((clen-prad)/plen)
      delta=delta*180/pi
      alpha=delta*sin(theta*pi/180.)
      beta=delta*cos(theta*pi/180.)
*****

```

```

*      output write statement
*****
      write(2,102)j,delta,theta,alpha,beta,psi
      read(1,*)
      j=j+1
      goto5

*
*-----
*
6      close(1)
      close(2)
101    format(a)
102    format(' ',i3,5f12.3)
103    format(' ',//,2x,'no.',6x,'delta',5x,'theta',8x,
:      'alpha',7x,'beta',8x,'psi',/)
      end

*
*****
*
*This subroutines does the preliminary calculations required to
*determine the angle between two lines.
*
      subroutine  angle(x1,y1,x2,y2,ca,cb)
      dist=sqrt((x2-x1)*(x2-x1)+(y2-y1)*(y2-y1))
      ca=(x2-x1)/dist
      cb=(y2-y1)/dist
      end

*
*****
*
*This subroutine computes the center of the fin-blade imprints.
*
      subroutine circen(x,y,numpts,x0,y0,rad)
      double precision c(2,2),r(2)
      dimension x(10),y(10)

```

```

6      do 1 i=1,2
      r(i)=0.
      do 1 j=1,2
1      c(i,j)=0.

      do 4 i=1,numpts
      t1=(2*x0-2*x(i))
      t2=(2*y0-2*y(i))
      t3=rad*rad- ((x(i)-x0)*(x(i)-x0) +(y(i)-y0)*(y(i)-y0))

      c(1,1)=c(1,1)+t1*t1
      c(2,2)=c(2,2)+t2*t2
      c(2,1)=c(2,1)+t1*t2

      r(1)=r(1)+t3*t1
4      r(2)=r(2)+t3*t2
      c(1,2)=c(2,1)

      call simul(c,r,2)
      count=count+1
      if (count.gt.100)return
      if(abs((r(1)).gt..001).or.(abs(r(2)).gt..001))goto5
      return
5      x0=x0+r(1)
      y0=y0+r(2)
      goto 6
      end

```

*

*

*This subroutine solves a system of equations.

*

```
      SUBROUTINE SIMUL(C,R,N)
      DOUBLE PRECISION TEMP,R(2),C(2,2)
      K=1
9      TEMP=1./C(K,K)
      J=K
2      C(K,J)=C(K,J)*TEMP
      IF(J.EQ.N)GOTO1
      J=J+1
      GOTO2
1      R(K)=R(K)*TEMP
      J=1
8      IF(K.EQ.J)GOTO3
      TEMP=C(J,K)
      L=K
5      C(J,L)=C(J,L)-C(K,L)*TEMP
      IF(L.EQ.N)GOTO4
      L=L+1
      GOTO5
4      R(J)=R(J)-R(K)*TEMP
3      IF(J.EQ.N)GOTO6
      J=J+1
      GOTO8
6      IF(K.EQ.N)GOTO7
      K=K+1
      GOTO9
7      RETURN
      END
```

```

*
*****
*
*This subroutines determines if the angle (phi or psi) is greater
*then 180 degrees. This is required because of how the arc
*cosine function is computed.
*

subroutine truang(xc,yc,yr2,xr2,yr1,xr1,xdelta,ydelta,phi)
real mref

if(xr2.eq.xr1)then

if(xc.gt.xdelta)phi=360.-phi

else

mref=(yr2-yr1)/(xr2-xr1)
bref=yc-mref*xc
ck=mref*xdelta+bref
if(mref.ge.0)then

if(ydelta.gt.ck)then
phi=360.-phi
endif

else

if(ydelta.lt.ck)then
phi=360.-phi
endif

endif

endif
end

```

APPENDIX B:
INSTRUCTIONS FOR USING THE YAW-CARD REDUCTION PROGRAM

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The following instructions and example are provided to aid in the use of the program listed in Appendix A.

STEP 1. Create "RDTYPE.DAT" file

This file contains information about the projectile.
The file has the following lines:

DESCRIB: A one-line description of the round
FINSP: The diameter (or span) of the fin blades
PLEN: length of the cylindrical portion of the projectile
PRAD: radius of the projectile
FINNO: number of fin blades

Figure B1 shows an illustration of the above variables.
The following is an example listing of a "RDTYPE.DAT" file:

M900 projectile

3.

23.35

.46

6.

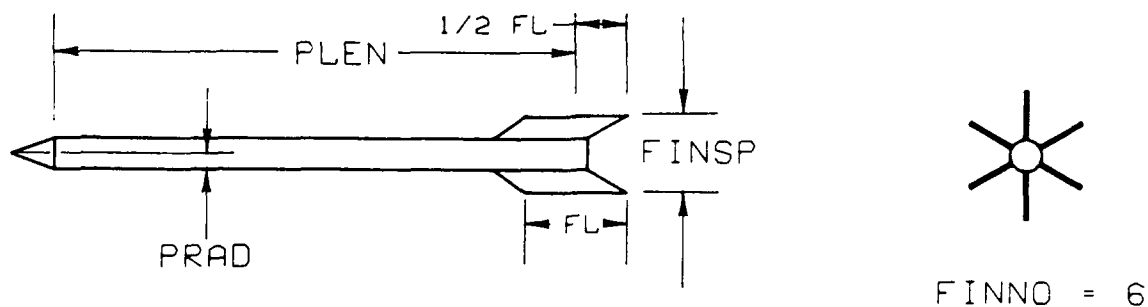


Figure B1. Projectile Information Required to Execute Program.

Note that the variables *FINSP*, *PLEN* and *PRAD*, must have the same unit of measure. In this example, the unit is inches. Also note that *FINNO* is the number of fin blades, not the number of fin blade imprints on a particular yaw card.

A new "RDTYPE.DAT" file must be created whenever a different type of projectile is use.

STEP 2. Digitizing Pad Layout

The layout for the digitizing pad is shown in Figure B2.

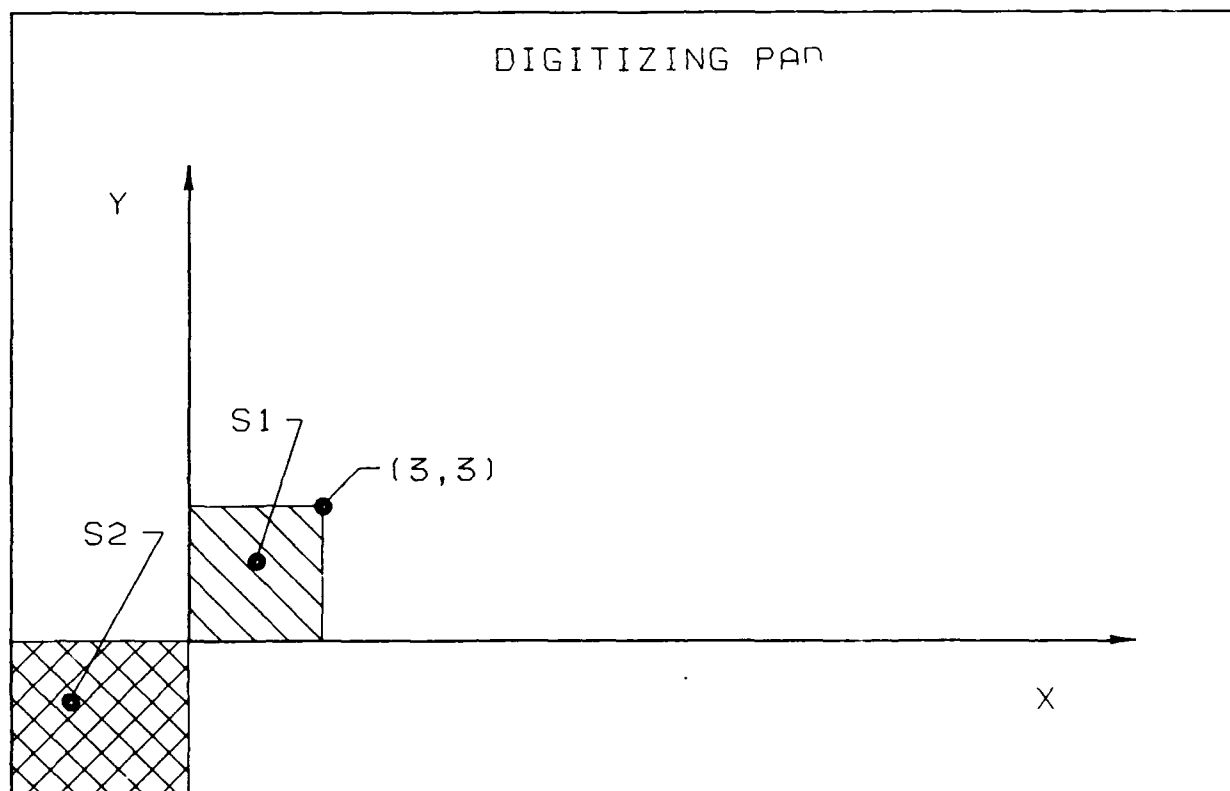


Figure B2. Digitizing Pad Layout.

The Areas S1 and S2 are "switches" required by the program. S1 signals the program that all the usable fin blade imprints are entered. This switch is required because there are cases when the round (because of the yawing motion) removes the material on the yaw card where a fin-blade imprint may occur. This creates a missing fin-blade imprint. The S2

imprint signals the program that all the data from one yaw card are entered. This switch is provided for user convenience. It allows the user to review a file with digitized data and see where the data from one yaw card ends and the data of another card begins.

STEP 3. "Digitizing" a Yaw Card

The points that need to be digitized are shown in Figure B3.

The digitized data goes in a file called "DIGDAT.DAT" in the following order:

Q1-Q2

Reference line data. $Q1_y > Q2_y$ and Q1 must be entered first.

Q3-Q8

Extreme edges of the fin blade imprints. They can be entered in any order.

Q9

This creates a flag in the data file that signals the program that all the fin imprints are entered.

Q10

Approximate center of fin imprints. Required to start iteration process to find true center of the imprints.

Q11

Extreme edge of the yaw imprint.

Q12

This creates a flag in the data file to mark the end of the data for this yaw card.

The following is an example of a "DIGDAT.DAT" file:

4.58343	5.71357
4.98731	1.57662
5.22843	4.84069
5.53013	6.27565
6.92620	6.74218
8.01478	5.74371
7.73103	4.32088
6.33700	3.84737
2.10548	0.86783
6.74561	5.32114
7.45783	5.31157
-1.34795	-1.49891

STEP 4. Execution of Program

With the files "RDTYPE.DAT" and "DIGDAT.DAT" created, no other information is required by the program. The program will compute δ , α , β , θ and ψ when the program is executed.

A sample from the output file "OUTPUT.DAT" is shown below:

M900 projectile

no.	delta	theta	alpha	beta	psi
1	0.919	184.370	-0.070	-0.916	47.576

where no. is the yaw card number.

Although only one yaw card result is shown in this example, several yaw cards' digitized data can be in the same "DIGDAT.DAT" file. They must be placed in that file in the proper sequence (as describe previously with the "Q" points) and in the proper yaw-card order, the first yaw-card's data in the beginning of the file and the last yaw-card's data at the end of the file.

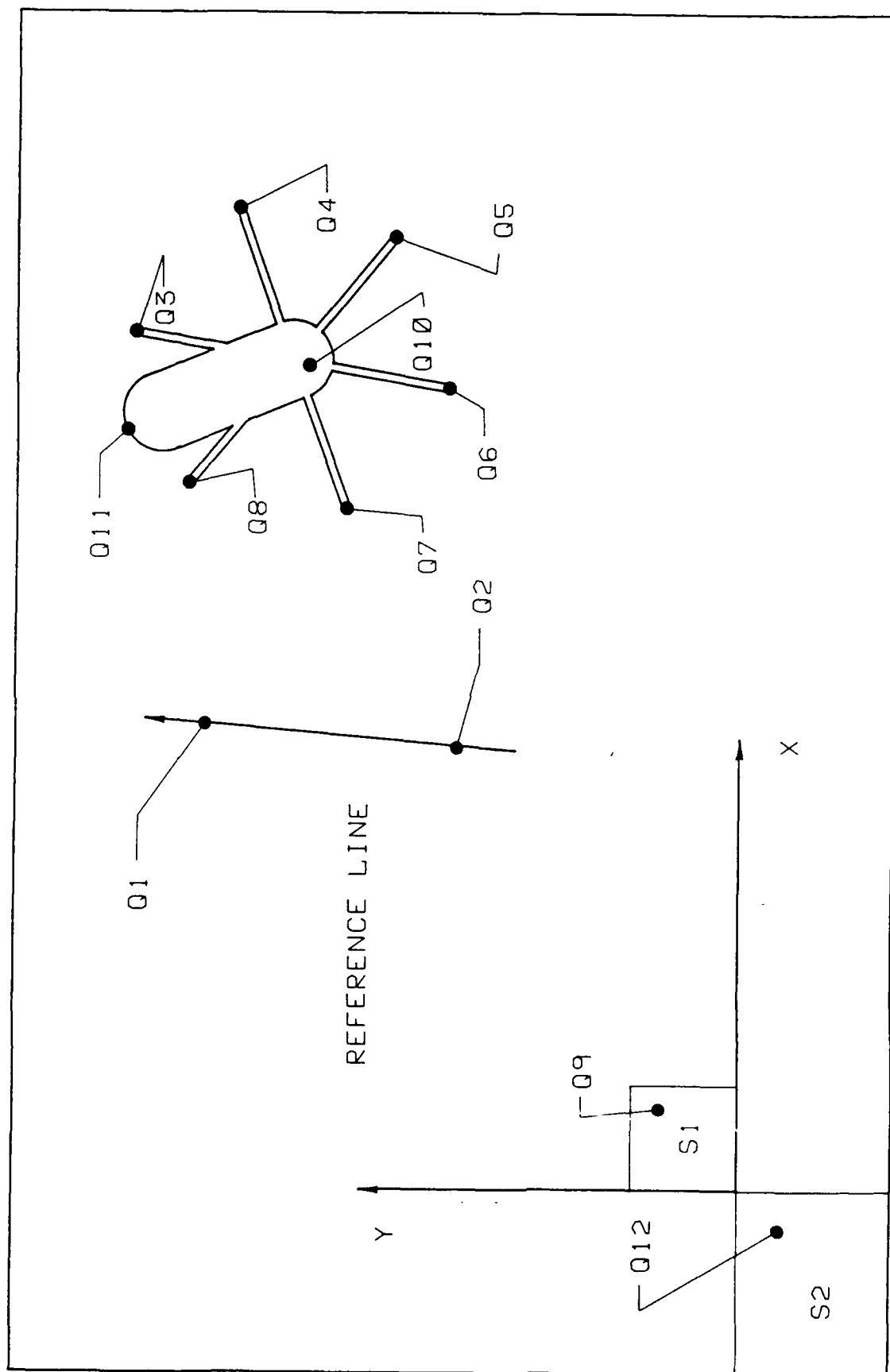


Figure B3. Required Digitized Data to Execute Program.

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APPENDIX C:
HOW THE ℓ_p LENGTH IS DETERMINED

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The length ℓ_p is required to determine δ . As seen in Figure C1.

$$\ell_p = \ell_t - Q, \quad (C1)$$

where ℓ_t is the largest distance between the beginning of the cylindrical portion of the rod and the location on the projectile where the fin blades are at their largest span. A study was conducted to determine how the Q length is best determined. It was assumed that the center of the fin-blade imprints was known.

The first case examined is a projectile that has two fin blades in the yaw plane. This is shown in Figure C1. When this occurs, the length of the cut made by the fin blades is $2L$. The location of the vertical center of the fin blade imprints is L and as stated previously, the L value is assumed known. The intersection of lines l_{fc} and l_{pc} (the centerline of the flight body) is required to determine Q .

The Q length is

$$Q = N + M, \quad (C2)$$

where

$$N = (E + \frac{1}{2}D)\tan\delta \quad (C3)$$

and for any nonzero δ of interest ($0^\circ < \delta \leq 10^\circ$)

$$M = \frac{I}{\sin\delta}. \quad (C4)$$

The I value is obtained by

$$I = L - \frac{E + \frac{1}{2}D}{\cos\delta}, \quad (C5)$$

and L is

$$L = \frac{1}{2}(F + G + H + J + K). \quad (C6)$$

It can be shown that

$$F = A\sin\delta, \quad (C7)$$

$$G = E\cos\delta, \quad (C8)$$

$$H = D\cos\delta, \quad (C9)$$

$$J = E\cos\delta, \quad (C10)$$

and

$$K = B\sin\delta. \quad (C11)$$

When the terms are combined,

$$I = \frac{A + B}{2}\sin\delta - (E + \frac{1}{2}D)\sin\delta\tan\delta \quad (C12)$$

and

$$M = \frac{1}{2}(A + B) - (E + \frac{1}{2}D)\tan\delta. \quad (C13)$$

Finally, from Equations C2, C3 and C13

$$Q = \frac{1}{2}(A + B). \quad (C14)$$

The second case examined is a projectile with a two-bladed fin, where the blades are in the plane perpendicular to the yaw plane. This is illustrated in Figure C2. In this case, the center of the fin blade imprints is

$$L = \frac{1}{2}(A + B)\sin\delta. \quad (C15)$$

Again, the location of the intersection of lines l_{fc} and l_{pc} is required to determine Q . The length Q is

$$Q = \frac{L}{\sin\delta}; \quad (C16)$$

therefore,

$$Q = \frac{1}{2}(A + B). \quad (C17)$$

The results of these two examples are the same. It is assumed that if the center of the fin imprints (L in Figures C1 and C2) can be determined, the correct Q length for all other cases is

$$Q = \frac{1}{2}(A + B). \quad (C18)$$

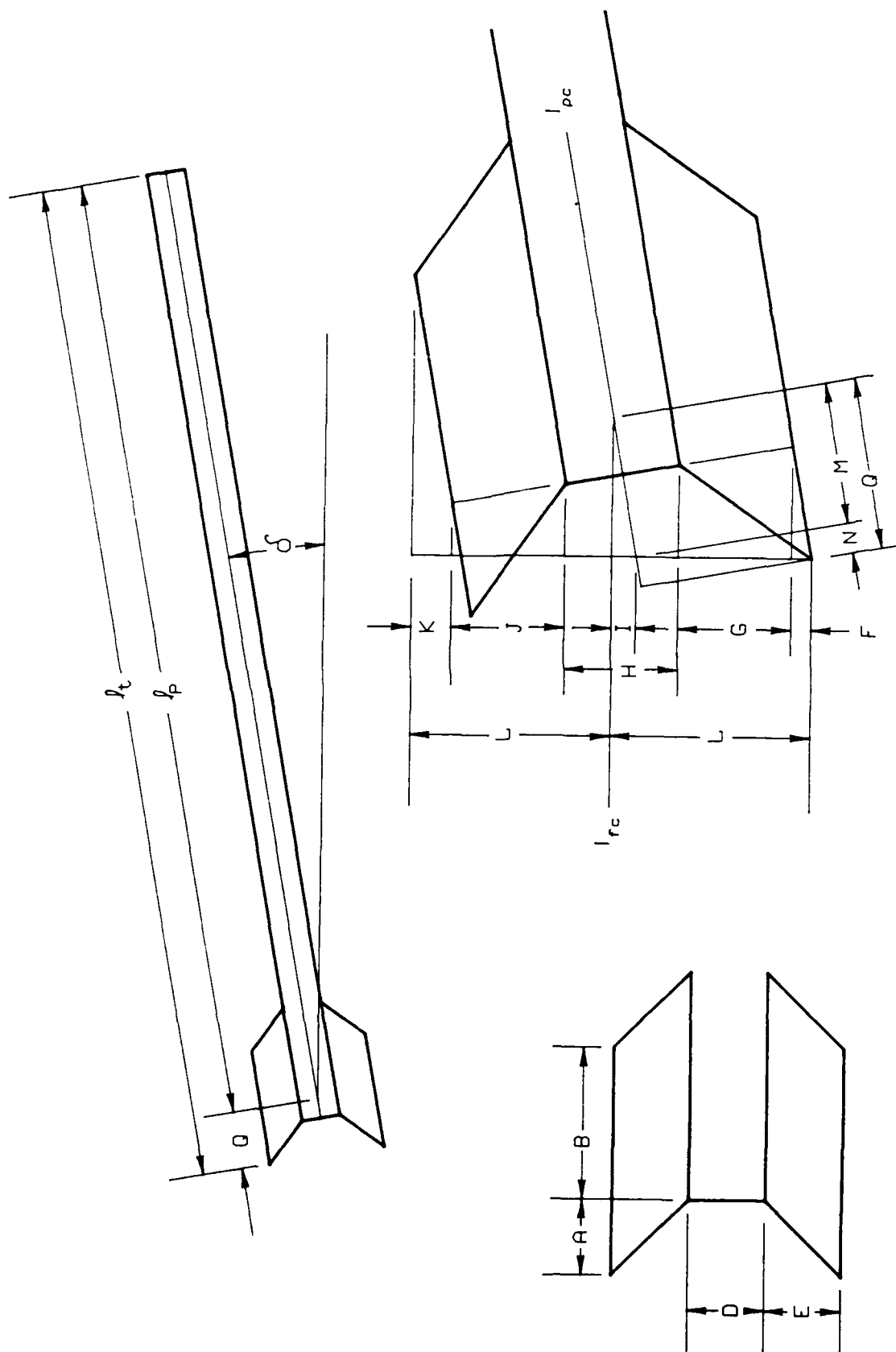


Figure C1. Case 1: Fin Blades in Yaw Plane.

LIST OF SYMBOLS AND ABBREVIATIONS

A_1	and B_1 are two of the direction cosines of the line segment that has Points P1 and P2 (as seen in Figure 3) as end points. A_2 and B_2 are similarly defined for the reference line.
b_{ref}	y-intercept of the line that has a slope of Δ_{ref} and goes through the center of the fin blade imprints.
C_{len}	distance between the extreme edge of the yaw-card hole and the center of the fin blade imprints
ℓ_p	projectile length required to determine δ . See Figure B1.
ℓ_t	largest distance between the beginning of the cylindrical portion of the rod and the location on the projectile where the fins are at their largest span. See Figure B1.
Pn_x	the "x" value of the n^{th} P point, where the P points are specific points on a yaw card. See Figure 3.
Pn_y	the "y" value of the n^{th} P point, where the P points are specific points on a yaw card. See Figure 3.
Qn	points on the yaw card that must be digitized in order to execute the program in Appendix A.
Q	$\ell_t - \ell_p$; it is $\frac{1}{2}$ FL on Figure B1.
r	the largest radius of the flight body
R_{fin}	radius of the fin-span circle. It is FINSP in Figure B1.

Greek Symbols

α	pitch
β	yaw
δ	total angle of attack
Δ_{ref}	slope of the reference line in the digitizer's coordinate system
ϕ	angle between the reference line and the line that contains Points P1 and P2 (see Figure 3). The angle begins at the reference line and goes in a clockwise direction.
ϕ^*	principle value of the arc cosine function
ψ	angle between the β -axis and the first fin blade imprint clockwise of the β -axis
θ	angle between the β -axis and the line that contains Points P1 and P2 (see Figure 3). The angle begins at the β -axis and goes in a clockwise direction.
θ_c	windsheild angle

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